TRANSIENT MASS TRANSFER AT A PLATE AND AT THE FRONTAL STAGNATION POINT IN A LONGITUDINAL STREAM OF A NONLINEARLY VISCOUS FLUID

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Results are shown of a theoretical study concerning the transient convective mass transfer at a semiinfinitely large plate and in the frontal stagnation zone in a stream of a nonlinearly pure-viscous fluid with a power-law concentration gradient at the plate surface.

In a study of the transient convective heat and mass transfer in a stream of linearly pure-viscous fluids [1] it has been shown that at a Prandtl number Pr ≫ 1 the processes of heat and mass transfer develop in a far from quasisteady mode. We will now analyze these processes under conditions of steady flow of an incompressible non-Newtonian fluid with a power-law rheological equation of state [3]:

$$\tau_{ij} = -p\delta_{ij} + k\left|\frac{1}{2}\dot{l}_{rm}\dot{l}_{mr}\right|^{\frac{n-1}{2}}. \tag{1}$$

The original equations of the boundary layer are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = V\frac{\partial V}{\partial x} + \frac{k}{\rho}n\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$\frac{\partial c_1}{\partial t} + u\frac{\partial c_1}{\partial x} + v\frac{\partial c_1}{\partial y} = D\frac{\partial^2 c_1}{\partial y^2}, \tag{4}$$

where $V = bx^i$ with $i = 0, 1$ for a plate and for the frontal stagnation point, respectively.

It is assumed here that the parameters in the rheological equation of state (1) as well as the physical properties do not depend on the concentration. The mass transfer is considered weak, in the sense that the concentration field of the diffusing impurity does not affect the dynamic flow characteristics. The boundary conditions for the dynamic problem are here

$$u(x, 0) = v(x, 0) = 0; \quad u(x, \infty) = V. \tag{5}$$

When the concentration gradient at the body surface varies according to a power law, then the initial and the boundary conditions for the equation of mass transfer are

$$c_1(x, y, 0) = c_0; \quad c_1(x, 0, t) = 0; \quad c_1(x, \infty, t) = c_0. \tag{6}$$

For integrating the system (4), (6) we will use the so-called similarity solutions. The velocity components and the self-adjoint variables are [8]

$$u = VF'(\eta), \tag{7}$$

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for a plate

$$v = x^{-\frac{n}{1+n}} (\eta F' - F) \frac{1}{1+n} [n(1+n) k/\rho V^{2n-1}]^{\frac{1}{1+n}}, \tag{8}$$

$$\eta = y x^{-\frac{1}{1+n}} \left[\frac{V^{2-n}\rho}{n(1+n)k}\right]^{\frac{1}{1+n}} = y x^{-\frac{1}{1+n}} M^{\frac{1}{1+n}}, \tag{9}$$

and for the frontal stagnation point

$$v = -\frac{2bn}{1+n} x^{\frac{n-1}{n+1}} \left[\frac{2\rho b^{2-n}}{k(1+n)}\right]^{-\frac{1}{1+n}} \left(F - \frac{1-n}{2n}\eta F'\right), \tag{8'}$$

$$\eta = y x^{\frac{1-n}{1+n}} \left[\frac{2\rho b^{2-n}}{k(1+n)}\right]^{\frac{1}{1+n}} = y x^{\frac{1-n}{1+n}} M^{\frac{1}{1+n}}. \tag{9'}$$

We change from physical coordinates x, y, t to coordinates x, Ψ, t. Then

$$\frac{\partial c_1}{\partial t} + u\frac{\partial c_1}{\partial x} = u\frac{\partial}{\partial \Psi}\left[uD\frac{\partial c_1}{\partial \Psi}\right], \tag{10}$$

$$c_1(x, \Psi, 0) = c_0; \quad c_1(x, 0, t) = 0; \quad c_1(x, \infty, t) = c_0, \tag{11}$$

where $u = \partial\Psi/\partial y$; $v = -\partial\Psi/\partial x$, and the flow function Ψ is defined by the relation

$$\Psi = x^{\frac{1}{1+n}} V M^{-\frac{1}{1+n}} F(\eta) \text{ for a plate}, \tag{12}$$

$$\Psi = x^{\frac{n-1}{n+1}} V M^{-\frac{1}{1+n}} F(\eta) \text{ for the frontal stagnation point.} \tag{12'}$$

High values of the Prandtl number, characteristic of diffusion processes in liquids, allow us to use a linear approximation for the velocity at a wall:

$$\frac{u}{V} = \left[\frac{\partial(u/V)}{\partial\eta}\right]_{\eta=0} \eta = a\eta, \tag{13}$$

where the values of $a = [\partial(u/V)/\partial\eta]_{\eta=0}$ for various n have been taken from the monograph [8]. The quantities u, x, and Ψ for a plate are related as follows [9]:

$$\eta = [2a^{-1} x^{-\frac{1}{1+n}} V^{-1} M^{\frac{1}{1+n}} \Psi]^{1/2}, \tag{14}$$

$$u = [2a x^{-\frac{1}{1+n}} V M^{\frac{1}{1+n}} \Psi]^{1/2}. \tag{15}$$

Analogously, for the frontal stagnation point we can obtain

$$\eta = [2a^{-1} x^{\frac{1-n}{1+n}} V^{-1} M^{\frac{1}{1+n}} \Psi]^{1/2}, \tag{14'}$$

$$u = [2a x^{\frac{1-n}{1+n}} V M^{\frac{1}{1+n}} \Psi]^{1/2}. \tag{15'}$$

With the aid of (15), (15'), and the dimensionless quantities

$$c = \frac{c_0 - c_1}{c_0}; \quad \tau = \frac{B^{4/3} t}{x^H D} = \frac{Vt}{x} \mathrm{Pr}_x^{-1/3} A^{2/3}; \quad \omega = \frac{\Psi^{1/2}}{x^E \left(\frac{3}{4}\cdot\frac{B}{E}\right)^{1/3}}, \tag{16}$$

where

$$\mathrm{Pr}_x = \frac{Vx}{D}\left(\frac{V^{2-n}\rho x^n}{k}\right)^{-\frac{2}{1+n}} = \mathrm{Pe}_x \mathrm{R}_x^{-\frac{2}{1+n}} \tag{17}$$

is the universal Prandtl diffusion number,

$$A = \left\{\frac{a(2n+1)/(n+1)}{[n(n+1)]^{\frac{1}{1+n}} 18}\right\}; \quad E = \frac{2n+1}{6(n+1)}; \quad H = \frac{4+2n}{3(n+1)} \tag{18}$$

for a plate

$$A = \left\{\frac{a(2+n)}{9(n+1)}\left(\frac{2}{1+n}\right)^{\frac{1}{1+n}}\right\}; \quad E = \frac{2+n}{3(1+n)}; \quad H = \frac{2n-2}{3(1+n)} \tag{18'}$$

for the frontal stagnation point, and

$$B = [2abM^{\frac{1}{1+n}}]^{1/2} \cdot D, \tag{19}$$

we transform (10) and (11) into

$$\frac{\partial c}{\partial \tau} - \frac{3H}{E}\omega\tau\frac{\partial c}{\partial \tau} - 3\omega^2\frac{\partial c}{\partial \omega} = \frac{\partial^2 c}{\partial \omega^2}, \tag{20}$$

$$c(\omega, 0) = 0; \quad c(0, \tau) = 1; \quad c(\infty, \tau) = 0. \tag{21}$$

We will solve system (20)-(21) by the method first shown in [4] and developed further in [5, 7], where its high accuracy was an important consideration.

A Laplace transformation

$$\bar{c}(\omega, s) = \int_0^\infty \exp(-s\tau)\, c(\omega, \tau)\, d\tau \tag{22}$$

changes system (20)-(21) into

$$\bar{c}'' + 3\omega^2\bar{c}' = \frac{3H}{E}\omega\frac{\partial(s\bar{c})}{\partial s} + s\bar{c}, \tag{23}$$

$$\bar{c}(0) = \frac{1}{s}; \quad \bar{c}(\infty) = 0. \tag{24}$$

We will seek the solution which satisfies system (23)-(24) in the form

$$\bar{c} = \frac{1}{s}\exp\left\{-\frac{\omega^3}{2} + \frac{H}{4E}\omega^3 - (s+\lambda)^{1/2}\omega\right\}\sum_{m=0}^{\infty} g_m(\omega)(s+\lambda)^{-m/2}, \tag{25}$$

where $\operatorname{Re} s > 0$ and λ is still an unknown positive function of ω. We let $g_0 \equiv 1$ and $g_1(0) = g_2(0) = \ldots = 0$. Then $\bar{c}(0) = 1/s$ and, as will be shown later, $\bar{c}(\infty) = 0$. The unknown function λ will be sought from the solution to the steady-state problem. An introduction of this function renders the series (25) applicable at any instant of time.

Inserting (25) into (23) and equating the coefficients in the terms of like powers in $(s + \lambda)$, we find for $m \geq 1$

$$\begin{aligned} g'_m = {} & \frac{g''_{m-1}}{2} + \frac{3H}{4E}\omega^2 g'_{m-1} - \frac{1}{2}\omega\lambda' g'_{m-2} - \frac{m-3}{2}\lambda' g'_{m-3} \\ & + \left\{-\frac{9}{8}\omega^4 + \frac{9H^2}{32E^2}\omega^4 - \frac{3H}{4E}\omega m - \frac{3\omega}{2} + \frac{\omega\lambda'}{2} + \frac{\lambda}{2}\right\} g_{m-1} \\ & + \left\{-\frac{3H}{8E}\omega^3\lambda' + \frac{1}{2}\lambda'(m-3) - \frac{1}{4}\omega\lambda'' - \frac{3H}{4E}\omega^2\lambda\right\} g_{m-2} \\ & + \left\{\frac{1}{8}\omega^2(\lambda')^2 - \frac{3H}{8E}\omega^2\lambda'(m-3) - \frac{1}{4}(m-3)\lambda'' \right. \\ & \left. - \frac{3H}{4E}\omega\lambda(m-3)\right\} g_{m-3} + \frac{1}{8}\omega(\lambda')^2(2m-7)\, g_{m-4} \\ & + \frac{1}{8}(m-3)(m-5)(\lambda')^2 g_{m-5}, \end{aligned} \tag{26}$$

$$g_{-1} = g_{-2} = g_{-3} = g_{-4} = 0.$$

The expression for the concentration field of the boundary layer is found from (25) by an inverse Laplace transformation:

$$c(\omega, \tau) = \exp\left\{-\frac{\omega^3}{2} + \frac{H}{4E}\omega^3 - \lambda^{1/2}\omega\right\}\sum_{m=0}^{\infty} g_m(\omega)\,\lambda^{-m/2}\, G_m, \tag{27}$$

where $G_0 = f_1 + f_2$ and $G_1 = f_1 - f_2$;

$$\underset{m \geq 2}{G_m} = 2^{m-2}\exp(\lambda^{1/2}\omega)\int_0^{\lambda\tau} z^{m/2-1}\exp(-z)\, i^{m-2}\operatorname{erfc}\left(\frac{1}{2}\lambda^{1/2}\omega z^{-1/2}\right) dz, \tag{28}$$

every G_m is expressed in terms of four functions

$$f_1 = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\omega\tau^{-1/2} - (\lambda\tau)^{1/2}\right];$$

$$f_2 = \frac{1}{2}\,[\exp(2\lambda^{1/2}\omega)]\,\mathrm{erfc}\left[\frac{1}{2}\omega\tau^{-1/2} + (\lambda\tau)^{1/2}\right];$$

$$f_3 = [\exp(\lambda^{1/2}\omega - \lambda\tau)]\,\mathrm{erfc}\left(\frac{1}{2}\omega\tau^{-1/2}\right); \tag{29}$$

$$f_4 = (2\pi)^{-1/2}(\lambda\tau)^{1/2}\exp\left\{-\left[\frac{1}{2}\omega\tau^{-1/2} - (\lambda\tau)^{1/2}\right]^2\right\},$$

and g_m is found by integrating the recurrence relation (26) beginning with m = 1.

Function G_m has the following properties [5, 7]:

1. For $0 < \tau < \infty$ and $0 < \omega < \infty$ it satisfies the inequality

$$0 < G_m < 1; \tag{30}$$

2. $\lim\limits_{\tau\to\infty} G_m = 1$.

At $\tau \to \infty$ expression (27) reduces to

$$c_w(\omega) = \exp\left\{-\frac{\omega^3}{2} + \frac{H}{4E}\omega^3 - \lambda^{1/2}\omega\right\}\sum_{m=0}^{\infty} g_m(\omega)\,\lambda^{-m/2}, \tag{31}$$

which yields the steady-state concentration field of the boundary layer. We are further interested in the local diffusion current flowing to a wall.

Differentiating (27) with respect to ω and then letting $\omega = 0$, we obtain

$$-c'(0,\tau) = \left\{\frac{\exp(-\lambda\tau)}{(\pi\tau)^{1/2}} + \lambda^{1/2}\,\mathrm{erf}\,(\lambda\tau)^{1/2}\right\} - \sum_{m=0}^{\infty}\frac{g'_m(0)}{\lambda^{m/2}}\cdot\frac{\Gamma_{\lambda\tau}(m/2)}{\Gamma(m/2)}, \tag{32}$$

where λ is $\lambda(0)$, and

$$g'_0(0) = 0;\quad g'_1(0) = \frac{\lambda}{2};\quad g'_2(0) = \frac{3H}{8E} - \frac{3}{4};\quad g'_3(0) = \frac{\lambda^2}{8};$$

$$g'_4(0) = \frac{3H}{8E}\lambda - \frac{3}{4}\lambda;\quad g'_5(0) = \frac{117H^2}{64E^2} - \frac{225H}{64E} - \frac{9}{32} + \frac{\lambda^3}{16};$$

$$g'_6(0) = \frac{3H}{8E}\lambda^2 - \frac{3}{4}\lambda^2; \tag{33}$$

$$g'_7(0) = \frac{585H^2}{128E^2}\lambda - \frac{1125H}{128E}\lambda - \frac{45}{64}\lambda + \frac{5}{128}\lambda^4;$$

$$g'_8(0) = \frac{8910H^3}{256E^3} - \frac{17415H^2}{256E^2} - \frac{810H}{64E} + \frac{1215}{64} + \frac{3H}{8E}\lambda^3 - \frac{3}{4}\lambda^3\ldots$$

are obtained by successive differentiations of (26). At $\tau \to \infty$ (32) reduces to

$$-c'_w(0) = \lambda^{1/2}(0) - \sum_{m=0}^{\infty}\frac{g'_m(0)}{\lambda(0)^{m/2}}. \tag{34}$$

For finding $\lambda(0)$, one must know the solution to the steady-state problem. It is given in [8, 9]:

$$c_w = \left[\frac{1}{3}\Gamma(1/3)\right]^{-1}\int_{\omega}^{\infty}\exp(-z^3)\,dz, \tag{35}$$

$$-\left.\frac{\partial c_w}{\partial\omega}\right|_{\omega=0} = \left[\frac{1}{3}\Gamma(1/3)\right]^{-1}. \tag{36}$$

Equating (34) and (36), we find $\lambda(0)$ for the respective values of parameter n. The results of calculations are listed in Table 1.

TABLE 1. Values of $\lambda^{1/2}(0)$ for Various Values of Parameter n

$\lambda^{1/2}(0)$	n					
	1,5	1,2	1	0,8	0,6	0,4
Plate	3,02	3,053	3,081	3,117	3,163	3,227
Stagnation point	2,439	2,389	2,345	2,290	2,215	2,095

It must be noted that the series on the right-hand side of (34) is semidivergent and its sum is found with the aid of an Euler transformation [10], inasmuch as such a transformation improves its convergence.

Exactly in a similar way, equating (31) and (35), one can find $\lambda(\omega)$ for the respective values of parameter n. The series on the right-hand side of (31) can be either convergent or semidivergent. In the latter case, for finding its sum, it is also necessary to apply the Euler transformation.

The behavior of the series in (32) is noteworthy. The procedure used for determining $\lambda(0)$ implies clearly that the series in (34) tends toward a limit and that the ratio $\Gamma_{\lambda\tau}(m/2)/\Gamma(m/2)$ is always positive but smaller than unity at any finite time τ. Consequently, every term of the series in (32) is smaller than the corresponding term of the series in (34). Thus, the series in (32) must also have a limit. This series can be either convergent or semidivergent. The same applies also to the series in (27), if one considers the properties of function G_m.

Obviously, in (35) $\lim\limits_{\omega\to\infty} c_W = 0$ and, since Re s > 0, it follows from (25) and (31) that $\lim\limits_{\omega\to\infty}\bar{c}(\omega) = 0$, as has been indicated earlier. With the numerical value of $\lambda(0)$ known, one can calculate the local diffusion current for the respective values of, according to the relation

$$\mathrm{Nu}_x = -x\left(\frac{\partial c}{\partial y}\right)_{y=0} = \mathrm{R}_x^{\frac{1}{1+n}}\,\mathrm{Pr}_x^{1/3}A^{1/3}$$

$$\times\left\{\frac{\exp(-\lambda\tau)}{(\pi\tau)^{1/2}} + \lambda^{1/2}\,\mathrm{erf}\,(\lambda\tau)^{1/2} - \sum_{m=0}^{\infty}\frac{g_m'(0)}{\lambda^{m/2}}\cdot\frac{\Gamma_{\lambda\tau}(m/2)}{\Gamma(m/2)}\right\}, \tag{37}$$

where Pr_x is the universal Prandtl diffusion number defined by relation (17), $\mathrm{R}_x = V^{2-n}x^n/(k/\rho)$ is the universal Reynolds number, and A is defined by expressions (18)-(18'). From expressions (16), (17), and (37) follows that within the frontal stagnation zone at $n \neq 1$, unlike in the case of a Newtonian fluid [6, 11], the diffusion current is a function of the x coordinate. With $\exp(-\lambda\tau)$, $\mathrm{erf}\,(\lambda\tau)^{1/2}$, and $\Gamma_{\lambda\tau}(m/2)$ represented by series in small and large $\lambda\tau$, formula (37) reduces to

$$\mathrm{Nu}_x = \mathrm{R}_x^{\frac{1}{1+n}}\,\mathrm{Pr}_x^{1/3}A^{1/3}\left\{\frac{1}{(\pi\tau)^{1/2}} - \left(\frac{3H}{8E} - \frac{3}{4}\right)\tau - \frac{8}{\pi^{1/2}}\left(\frac{117H^2}{64E^2}\right.\right.$$

$$\left. - \frac{225H}{64E} - \frac{9}{32}\right)\tau^{5/2} - \frac{1}{24}\left(\frac{8910H^3}{256E^3} - \frac{17415H^2}{256E^2} - \frac{810H}{64E} + \frac{1215}{32}\right)\tau^4 \tag{38}$$

for small $\lambda\tau$ or

$$\mathrm{Nu}_x = \mathrm{R}_x^{\frac{1}{1+n}}\,\mathrm{Pr}_x^{1/3}A^{1/3}\left\{\lambda^{1/2} - \sum_{m=0}^{\infty}\frac{g_m'(0)}{\lambda^{m/2}} + \exp(-\lambda\tau)\frac{\lambda^{1/2}}{2\pi^{1/2}}\right.$$

$$\times\left(1 - \frac{3}{2\lambda\tau} + \cdots\right) + \exp(-\lambda\tau)\sum_{m=0}^{\infty}\frac{g_m'(0)}{\lambda^{m/2}}\,\frac{(\lambda\tau)^{m/2-1}}{\Gamma(m/2)}$$

$$\times\left[1 + \left(\frac{m}{2} - 1\right)\frac{1}{\lambda\tau} + \left(\frac{m}{2} - 1\right)\left(\frac{m}{2} - 2\right)\frac{1}{(\lambda\tau)^2} + \cdots\right] \tag{39}$$

for large $\lambda\tau$. As was to be expected, λ does not appear in (38). Initially, at $t \ll x\mathrm{Pr}_x^{1/3}/V\lambda(0)A^{2/3}$, the mass transfer process evolves in accordance with the laws of molecular diffusion and is determined by the first term of formula (38), which transforms into

$$\mathrm{Nu}_x = \mathrm{R}_x^{\frac{1}{1+n}}\,\mathrm{Pr}_x^{1/3}A^{1/3}\frac{1}{(\pi\tau)^{1/2}} = \mathrm{R}_x^{\frac{1}{1+n}}\,\mathrm{Pr}_x^{1/3}A^{1/3}\frac{x^{1/2}\,\mathrm{Pr}_x^{1/6}}{(\pi t)^{1/2}V^{1/2}A^{1/3}} = \mathrm{R}_x^{\frac{1}{1+n}}\,\mathrm{Pr}_x^{1/2}\frac{x^{1/2}}{(\pi t)^{1/2}V^{1/2}} = \frac{x}{(\pi tD)^{1/2}}. \tag{40}$$

Consequently, during the initial period of time Nu_x obviously does not depend on the rheological properties of the medium. During the latter period of time, at $t \gg x\mathrm{Pr}_x^{1/3}/V\lambda(0)A^{2/3}$, the steady-state mode of mass transfer stabilizes with

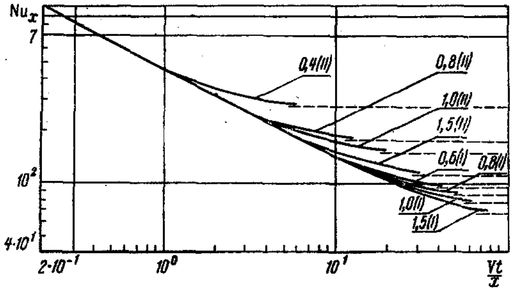

Fig. 1

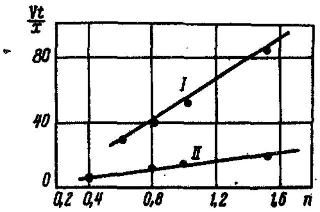

Fig. 2

Fig. 1. Transient convective mass transfer at (I) a plate and (II) in the frontal stagnation zone: V = 0.02 m/sec, x = 0.02 m, D = $0.625 \cdot 10^{-9}$ m^2/sec, k = $1.16 \cdot 10^{-3}$ kg · sec^{n-2}/m, $\rho = 1.015 \cdot 10^3$ kg/m^3. Dashed lines represent the steady state.

Fig. 2. Time of complete stabilization at (I) a plate and (II) in the frontal zone: V = 0.02 m/sec, x = 0.02 m, D = $0.625 \cdot 10^{-9} m^2$/sec, k = $1.16 \cdot 10^{-3}$ kg/sec^{n-2}/m, $\rho = 1.015 \cdot 10^3$ kg/m^3.

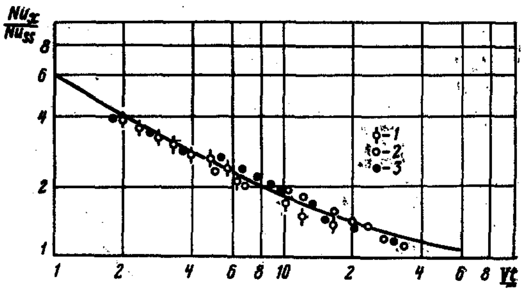

Fig. 3. Transient convective mass transfer at a plate (n = 1, Pr = $2 \cdot 10^3$, V = 0.0308 m/sec): 1) x = 0.0105 m and R_X = 228; 2) x = 0.0025 m and R_X = 54.3; 3) x = 0.0065 m and R_X = 14.1.

$$\mathrm{Nu}_x = \mathrm{R}_x^{\frac{1}{1+n}} \mathrm{Pr}_x^{1/3} A^{1/3} \left[\frac{1}{3} \Gamma(1/3) \right]^{-1}. \tag{41}$$

The characteristic stabilization time is determined according to the formula

$$T = \frac{x \,\mathrm{Pr}_x^{1/3}}{V \lambda(0) A^{2/3}}. \tag{42}$$

Calculations have shown that, with the other conditions unchanged, the stabilization time is longer for a plate than for the stagnation point. Thus, at

$$n = 0.4;\ V = 0.02\ \text{m/sec};\ x = 0.02\ \text{m};\ D = 0.625 \cdot 10^{-9}\ \text{m}^2/\text{sec},$$
$$k = 1.16 \cdot 10^{-3}\ \text{kg} \cdot \text{sec}^{n-2}/\text{m};\ \rho = 1.015 \cdot 10^3\ \text{kg/m}^3 \tag{43}$$

the characteristic time was T = 5.869 sec for the plate and T = 2.363 sec for the frontal stagnation point. Furthermore, with other conditions unchanged, the characteristic time decreases as the fluid becomes more pseudoplastic.

For a plate with the parameter values as specified in (43), the characteristic time according to (42) with n = 0.8 was T = 11.36 sec and with n = 0.4 it was T = 5.869 sec.

As the fluid becomes more dilatant, the characteristic time of stabilization increases. This peculiarity has been noted earlier in a study of mass transfer at the surface of a rotating disk [12].

Results of calculations according to (37) are shown in Fig. 1 in logarithmic coordinates, for a plate and for the frontal stagnation point, at various values of the non-Newtonian parameter n.

We will regard the stabilization to be complete at the time when the process has reached its steady state within 5%.

In Fig. 2 is shown the time of complete stabilization as a function of the parameter n, for a plate and for the frontal stagnation point.

It is quite evident that the process becomes steady first at the frontal stagnation point and then at the plate.

The ratio of the local transient current to the corresponding steady-state current is

$$\frac{j_x}{j_{xw}} = \frac{\mathrm{Nu}_x}{\mathrm{Nu}_{xw}} = \frac{1}{3} \Gamma(1/3) \left\{ \frac{\exp(-\lambda\tau)}{(\pi\tau)^{1/2}} + \lambda^{1/2} \operatorname{erf} (\lambda\tau)^{1/2} - \sum_{m=0}^{\infty} \frac{g'_m(0)}{\lambda^{m/2}} \frac{\Gamma_{\lambda\tau}(m/2)}{\Gamma(m/2)} \right\}. \tag{44}$$

TABLE 2. Comparison between the Local Transient Current Calculated for a Plate with n = 1 and the Results in [5, 2]

$\frac{Vt}{x}$	$Nu_x/Nu_{x,w}$		
	formula (44)	[5]	[2]
0,5	3,52	3,52	3,52
1	2,45	2,45	2,45
2	1,76	1,75	1,75
3	1,45	1,39	1,43
5	1,17	1,14	1,15
7	1,10	1,06	1,06
9	1,05	1,025	1,0

The results of calculations according to (44) for a plate are shown in Table 2, with n = 1 and Pr = 10, together with the data from [2, 5]. The curves overlap at low values of the dimensionless time parameter, but they separate by 1 to 6 or 7% at sufficiently high values of this parameter. At a high Prandtl number one would expect less discrepancy, inasmuch as the analysis in [2, 5] of the transient heat transfer was based on the total true velocity profile; in our analysis, on the other hand, the velocity profile of the diffusion boundary layer was approximated by a linear one, such an approximation being permissible at a high Prandtl number.

The theoretical data for a plate, based on (44) with n = 1 and $Pr = 2 \cdot 10^3$, are compared in Fig. 3 with experimental data according to [11]. An initial stepwise change of the concentration gradient at the wall and boundary conditions according to (6) were ensured during the test by electrochemical means.

NOTATION

c_1	is the concentration;
c_0	is the concentration in the stream;
x and y	are the space coordinates;
t	is the time;
Ψ	is the flow function;
u and v	are the velocity components along axes x and y, respectively;
V	is the velocity at the outer edge of the boundary layer;
D	is the diffusivity;
n	is the exponent characterizing the non-Newtonian behavior of a fluid;
k	is the consistency index;
η	is the self-adjoint variable;
τ, ω, and c	are dimensionless variables: time, distance, and concentration, respectively;
M	is a quantity defined by Eqs. (9), (9');
A	is a quantity defined by Eqs. (18), (18');
B	is a quantity defined by Eq. (19);
Pr_x	is the universal Prandtl diffusion number;
R_x	is the universal Reynolds number;
τ_{ij}	are components of the stress tensor;
i_{rm}, i_{mr}, and i_{ij}	are components of the strain tensor;
δ_{ij}	is the Kronecker delta.

LITERATURE CITED

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